

2.18.1. Semantic Problems: Validity

A. Translate each of the following arguments into formal language, then use **truth tables** to decide whether the argument is **valid**.

1. Lucretia didn't go to Novo unless she dyed her hair. Lucretia did go to Novo.
∴ Lucretia dyed her hair.
2. Either staple gun duels are illegal or cannon bowling's illegal. Cannon bowling is legal. ∴ Staple gun duels are legal.
3. Either staple gun duels are illegal or cannon bowling is. Cannon bowling is legal.
∴ Staple gun duels are illegal.
4. Kitty won't leave for Reno unless Dr. Slim does. ∴ Dr. Slim won't leave for Reno unless Kitty does.
5. Rex won't publish on both Business Logic and Quantum Cosmetology. He won't publish on Business Logic unless he publishes on Quantum Cosmetology.
∴ Rex won't publish on either Business Logic or Quantum Cosmetology.
6. Letitia and Lucretia won't both enjoy the movie. Letitia won't enjoy the movie.
∴ Lucretia will enjoy the movie.
7. Letitia and Lucretia won't both enjoy the movie. Lucretia will enjoy the movie.
∴ Letitia won't enjoy the movie.
8. Dick won't have a hot toddy unless Dora does. Dora won't have a hot toddy unless Dick does. ∴ Both Dick and Dora will have a hot toddy.
9. Dick won't have a hot toddy unless Dora does. Dora won't have a hot toddy unless Dick does. Either Dick won't have a hot toddy or Dora won't. ∴ Neither Dick nor Dora will have a hot toddy.
10. Either mahjongg and surfing are both legal or they're both illegal. Mahjongg isn't illegal. ∴ Mahjongg is legal and surfing is too.

11. We won't have both a tax cut and increased spending on Logic research. We won't have increased spending on logic research unless we have a tax cut too. \therefore We won't have increased spending on Logic research, but we will have a tax cut.

12. Trixie won't win the hand unless she does so without playing an ace. \therefore Unless she plays an ace, Trixie won't win the hand.

13. Either Barbie will celebrate Rex's birthday, or she'll celebrate it without giving him new a new bowling shirt. Unless she gives him a new bowling shirt, Barbie won't celebrate Rex's birthday. \therefore Barbie will celebrate Rex's birthday and give him a new bowling shirt.

14. Either Jake and Jezebel will both rejoin the cult or neither of them will. \therefore Jake won't rejoin the cult unless Jezebel does.

15. We're having truffles or grog, but not both. Either we aren't having truffles or we're having neither truffles nor grog. \therefore We're having grog without having truffles.

16. Either Trixie and Elvis will both work the Vegas tradeshow or neither of them will. Either Trixie or Elvis will fail to work the Vegas tradeshow. \therefore Either Trixie will work the Vegas tradeshow or Elvis will, but not both.

17. Jack isn't a bird that can fly. \therefore Either Jack isn't a bird, or Jack can't fly.¹

18. Jack isn't a bird that can fly. \therefore Jack isn't a bird unless he's one that can't fly.¹

19. Unless Dr. Slim is a physician, he's not a physician who performs surgery. Dr. Slim's not a physician. \therefore Dr. Slim doesn't perform surgery.¹

(See 2.20.1 Problem 6 for further discussion.)

20. Unless Dr. Slim is a physician who performs surgery, he's not a physician. Dr. Slim performs surgery. \therefore Dr. Slim is a physician.

21. Jake didn't study yet not pass the exam. Jake didn't pass the exam. \therefore Jake neither studied nor passed the exam.²

¹ See the remarks on relative clauses and negations in 2.10 §3.

² On the effect of deleted repetition in the first premise, see 2.9 §4.

22. Jake didn’t pass the exam without studying. Jake didn’t study. \therefore Jake neither studied nor passed the exam.³

23. Neko will complain unless we have tuna for dinner. Jack will complain unless we fail to have tuna for dinner. \therefore Either Neko will complain or Jack will.

24. Unless both Trixie and Elvis are going to Logicpalooza, Barbie isn’t going. Trixie’s not going to Logicpalooza, but either Elvis or Barbie is. \therefore Elvis is going to Logicpalooza.

25. Either Jack scaled the Cathedral of Learning but Barbie didn’t, or he para-surfed over Niagara Falls. Jack didn’t para-surf over Niagara Falls unless he scaled the Cathedral of Learning. \therefore Barbie didn’t scale the Cathedral of Learning.

26. Neko sold her flying toaster without testing it unless the safety inspector visited. Neko tested her flying toaster unless the safety inspector didn’t visit. \therefore Either Neko sold her flying toaster without the safety inspector visiting, or the safety inspector visited and Neko tested her flying toaster.

³ See the remarks on “without” sentences and negations in 2.10 §3.

B. Truth and Validity Puzzle. Suki, Neko, and Jack are on trial for the break-in at the Neptuna Seafood Lounge, and testify as follows.

Suki: Jack isn't guilty, but Neko is.

Neko: Suki isn't guilty unless Jack is.

Jack: I'm not guilty, but either Suki or Neko is.

Use truth tables to answer the following questions.

1. If everyone told the truth, who's guilty and who's not?
2. If no one is guilty, who gave false testimony?
3. If everyone is guilty, who gave false testimony?
4. One person's testimony follows validly from someone else's – whose?

(Adapted from Kleene 1967: 67, Problem 14.2; attributed to Jerome Keisler)

C. Truth and Validity Puzzle, Revisited. Suppose Suki, Neko, and Jack instead testify as follows.

Suki: Jack isn't guilty unless Neko is.

Neko: Suki isn't guilty, but Jack is.

Jack: I'm not guilty unless both Suki and Neko are.

Use truth tables to answer the following questions.

1. If everyone gave false testimony, who's guilty and who's not?
2. If no one is guilty, who gave false testimony?
3. If everyone is guilty, who gave false testimony?
4. One person's testimony follows validly from someone else's – whose?

D. Compare the answers to **Problem B** with the answers to **Problem C**. Can we explain their similarities and differences?

(See 2.33 and 2.34 for further discussion.)

E. Validity and Contradictory Conclusions. Call two sentences “**contradictory**” if one is the negation of the other – for example, “P” and “ $\sim P$ ”.

Explain why **only an inconsistent set of premises can entail contradictory conclusions**. (For example: explain why, **if both the following arguments are valid, Set of Sentences S must be inconsistent.**)

Set of Sentences S	Set of Sentences S
<hr/>	<hr/>
$\therefore P$	$\therefore \sim P$

F. Validity and Tautologies. Explain why a tautology only entails tautologies (that is: why the only kind of sentence that follows validly from a tautology is a tautology).

G. Validity and Contradictory Conclusions, Revisited. Explain why entailing contradictory conclusions is the hallmark of an inconsistent set of sentences – something **all and only** the inconsistent sets do.

Problem E made half of this case already, showing that entailing contradictory conclusions is something **only** an inconsistent set of sentences can do.

If a contradictory pair of sentences follows validly from Set of Sentences S, then S must be inconsistent.

Now strengthen that claim by explaining why **every** inconsistent set of premises entails contradictory conclusions.

If Set of Sentences S is inconsistent, then two contradictory sentences are bound to both follow validly from S.

H. Self-Negations. Explain why each of the following sentences is true.

1. Only a tautology follows validly from its own negation.
2. Only a contradiction entails its own negation.

(See 2.34.1 Problem A *for further discussion.*)